WASTELOAD ALLOCATION FOR CONSERVATIVE SUBSTANCES

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Chapter I
Introduction

Wasteload allocations for conventional parameters (primarily those affecting dissolved oxygen, D.O., concentration) are becoming an integral part of most state's permitting methodology. Used in conjunction with technology based limits, a wasteload allocation yields a uniformly applied permit which protects the beneficial uses designated for the receiving stream without undue economic hardship to the discharger.

The wasteload allocation for conventional parameters is based on the receiving stream's assimilative capacity. The Streeter-Phelps equation has been used in practically all important studies of stream assimilative capacity (Nemerow 1974). This insures that the wasteload allocation protects the water quality standard at every point in the receiving stream, since it protects it at the D.O. sag, where the D.O. concentration is at its minimum.

A logical extension of the wasteload allocation concept to conservative substances is being advocated by the Environmental Protection Agency (EPA). Eventually, wasteload allocation for conservative substances may be required in state's water quality management plans. Conservative substances are those which do not change rapidly when discharged to the aquatic environment, such as cadmium or copper, as opposed to the nonconservative parameters, which may change rapidly due to ambient conditions, such as D.O. or temperature.

A conservative substance wasteload allocation should yield a permit which protects the beneficial uses designated to the receiving stream. This is accomplished by constraining the concentration of the conservative substance in the discharge so that the numerical criteria for that substance are never exceeded in the region of the receiving stream where numerical criteria are applicable. Numerical criteria are usually specified for many common conservative substances in the water quality standards. The water quality standards are State regulations designed to protect the State's waters. When approved by EPA they become Federal regulation, and thus may be used in a wasteload allocation for a National Pollutant Discharge Elimination System (NPDES) permit, as well as for State permits.

In many states, the numerical criteria apply only outside a mixing zone (EPA 1980). Therefore, the numerical criteria cannot be used directly as permit limits. The mixing zone may be defined in many different ways, but usually a portion of the flow volume and/or cross sectional area is involved. For the purposes of this report, the mixing zone is defined as a portion of the flow volume within which the numerical criteria in the water quality standards do not apply. Several states reserve a portion of the flow volume as a mixing zone.

As the wasteload allocation for conventional parameters is based upon the assimilative capacity, the wasteload allocation for conservative substances is based upon the dilution capacity of the receiving stream. This is accomplished through the use of a dispersion model, which describes the dispersion of a substance in a fluid. The use of a dispersion model in the wasteload allocation allows more waste to be discharged to a large unpolluted river than to a smaller one, while still protecting the designated beneficial uses.

States are required to certify that NPDES permits do not violate water quality standards. This is relatively easy to do with a wasteload allocation which incorporates a dispersion model.

A dispersion model which may be used to develop a wasteload allocation will be derived. The model requires a minimum of input data, and the assumptions used in the derivation will be shown to be reasonable. A comparison of the dispersion model and a model developed using the mass balance assumption will be made.

Chapter 2
The Mass Balance Allocation

A dispersion equation which is commonly used in the wasteload allocation process for both conventional parameters and conservative substances is based on the mass balance assumption; that the pollutant concentration is uniformly distributed across the stream. The mass balance is expressed by (EPA 1986)

$$C_B Q_u + C_e Q_e = C(Q_u + Q_e) \qquad (1)$$

where C_B is the background concentration (pollutant coming down the river from upstream, assumed uniformly distributed in the stream), Q_α is the flow above the discharge, C_e is the effluent concentration (concentration of pollutant in the discharge) Q_e is the effluent discharge (flow), and C is the concentration of pollutant after the effluent has been completely mixed across the stream. Assuming $C_B = 0$, a wasteload allocation may be obtained from (1), with $C' \equiv C - C_B$ and $Q_e^* \equiv Q_e/Q_\alpha$.

$$\frac{C_e}{C'} = \frac{1 + Q_e^*}{Q_e^*} \qquad (2)$$

For the mass balance allocation, C may be defined not only as the concentration after complete mixing, but also as the numerical criteria in the water quality standards.

Figure 1 shows a plot of the ratio of the effluent concentration to the water quality standards vs. the ratio of the effluent to the upstream flow for the mass balance wasteload allocation (dotted line). Note that the dilution capacity of the stream is accounted for. When $Q_e^* = 1$, the effluent discharge equals the upstream flow and dilution is minimal. In this case the effluent concentration allowed by the wasteload allocation is only twice the standard ($C_e/C' = 2$). When $Q_e^* = 0.1$, the upstream flow is ten times that of the discharge, and the effluent concentration allowed by the wasteload allocation is eleven times the standard. This wasteload allocation enables the permitting authority to issue less stringent permits when dilution capacity is high, thus avoiding needless economic hardship.

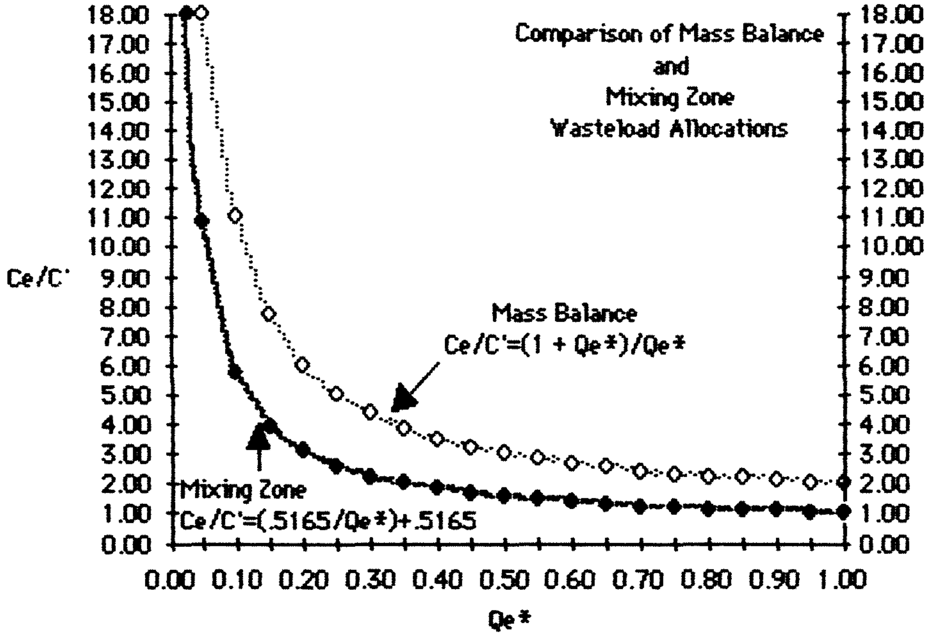


Figure 1. Comparison of mass balance (upper line) and mixing zone (lower line) wasteload allocations. Q_e^* is the ratio of the effluent discharge flow to the upstream dilution flow and, for negligible background concentrations ($C_B \approx 0$), C_e/C' is the ratio of the effluent concentration to the water quality standard.

Another advantage of the mass balance wasteload allocation is that it requires only a few readily available parameters which do not require special field trips to collect. Generally the seven day, ten year low flow value, or whatever low flow value is specified in the water quality standards, is used for Q_α. The effluent discharge, Q_e, is dictated by the production process and may be obtained from the permittee. The numerical criteria in the water quality standards may be used for C, and the background concentration, C_B, may either be determined from monitoring data, or assumed zero if no source of the permitted substance exists upstream.

Because State and Federal agencies place relatively little emphasis on permitting, minimal resources are available for wasteload allocation. Therefore, it is not feasible to use a dispersion model which requires more input data than mass balance does. Nor is a greater effort cost effective. The numerical criteria in the water quality standards are at best approximations of the maximum concentration of a particular substance a particular receiving stream can assimilate. This is one of the reasons numerical criteria are subject to periodic change. It makes no sense to expend a great deal of effort to collect the data necessary for sophisticated wasteload allocation when the numerical criteria in the water quality standards are themselves gross estimates of the maximum concentration the receiving stream can dilute without adversely impacting its beneficial uses.

Another advantage of the mass balance allocation is that it is not empirical. There is no reliance upon observations gathered at a few locations which are then assumed to have universal applicability. Wasteload allocation models for D.O. are empirical, and, because of this, generally have to be calibrated for each application. This is too expensive and time consuming to allow widespread use of a conservative substance wasteload allocation.

Perhaps the most important reason to use a wasteload allocation similar to mass balance for conservative parameters is that it largely eliminates arbitrary and capricious decision making by permitting authorities when issuing permits for conservative substances. Using a wasteload allocation provides a uniformity which is lacking when technology based permits are inappropriate. Probably the biggest incentives a regulatory agency has for adopting a wasteload allocation are economic and legal. Once a wasteload allocation process is widely accepted it saves agency resources. The allocation requires only minutes to compute by the permit drafter, and can save days of negotiations and hearings. After the allocation has been formally reviewed and accepted by the State and Federal governments, using it should insulate the permit drafter from legal action.

While there are many advantages to the mass balance approach, there are also shortcomings. One of the main disadvantages is that this wasteload allocation insures that numerical criteria will not be exceeded only after complete mixing has occurred, rather than everywhere that the numerical criteria are applicable. Complete mixing usually occurs far from the source. Therefore, a mass balance wasteload allocation will allow standards violations except where complete mixing has occurred.

Many States specify a zone of passage in their standards (EPA 1980). This usually is a flow volume or cross sectional area within the mixing zone where water quality standards apply. Since complete mixing has not occurred within the zone of passage, water quality standards will be violated in parts of this region if a mass balance wasteload allocation is used to permit the discharge. A viable wasteload allocation must produce a permit which requires the maximum concentration within the zone of passage to equal the numerical criteria.

Another problem with mass balance is verification. Since the point of complete mixing is unknown, concentration measurements have to be taken far downstream to ensure that complete mixing has occurred. At this point the signal to noise ratio may be so large that the effect of the discharge cannot be detected.

Chapter 3
Derivation of a Dispersion Model for Wasteload Allocation for Conservative Substances

The requirements for a viable wasteload allocation are now obvious. The input data must be limited to C_B, Q_α, Q_e and C (the water quality standard). The resulting discharge limitations must insure that the standard is not exceeded outside the mixing zone. This requires that the wasteload allocation be based on a physical law flexible enough to allow compliance with the standard. For this reason, the wasteload allocation developed here will be based on conservation of mass. The wasteload allocation cannot be empirical. Empiricism may require calibration for each receiving stream; this is beyond the capabilities of permitting agencies. The dispersion model derived below will be used to develop a wasteload allocation to satisfy these requirements.

Yotsukura and Sayre (1976) produced an elegant derivation of a diffusion equation, with virtually no assumptions which will serve as the foundation for the dispersion model. However, in order to obtain an analytical solution to this equation, so that only the four independent variables C_B, Q_α, Q_e and C are required, many assumptions are necessary. In order to show the consequences of each assumption, it will be made as early as feasible, rather than when actually needed.

In Cartesian coordinates the diffusion equation may be written:

$$\frac{\partial \theta}{\partial t} + \frac{\partial(\theta W_x)}{\partial x} + \frac{\partial(\theta W_y)}{\partial y} + \frac{\partial(\theta W_z)}{\partial z} = 0, \qquad (3)$$

where θ (x, y, z, t) is the instantaneous concentration and W_x, W_y, and W_z are stream flow speeds in the x (downstream along the bank), y (vertical) and z (transverse) directions.

To obtain an analytical solution to the diffusion equation, steady state conditions must exist. This means that not only do ambient conditions remain constant, but so do the effluent flow and concentration. Time dependence in (3) can be eliminated to obtain steady state conditions by time averaging. Using the overbar to denote averages,

$$\bar{\Phi} = \frac{1}{A}\int_{\alpha_1}^{\alpha_2} \Phi \, dx, \qquad (4)$$

where the interval $A \equiv \alpha_2 - \alpha_1$, is at least long enough to damp out turbulent fluctuations. Steady state conditions do not require parameters to remain constant forever; the critical parameters for our dispersion model are likely to be effluent flow and concentration. These parameters must remain steady for longer than the length of time required for the pollutant to traverse the distance from the source to the point of maximum concentration in the region water quality standards are applicable (i.e., the maximum concentration on the mixing zone boundary). This will generally be much longer than the time required to damp out turbulent fluctuations. Integration of (3) with respect to time yields

$$\int_{t_1}^{t_2} \left[\frac{\partial \theta}{\partial t} + \frac{\partial(\theta\, W_x)}{\partial x} + \frac{\partial(\theta\, W_y)}{\partial y} + \frac{\partial(\theta\, W_z)}{\partial z} \right] dt = 0 \qquad (5)$$

Integration can proceed term by term. The first term in (5) may be eliminated, since $\partial\theta/\partial t = 0$ under steady state conditions.

The second term in (5) may be rewritten using Leibnitz's rule

$$\int_{t_1}^{t_2} \frac{\partial(\theta W_x)}{\partial x}\, dt = \frac{\partial}{\partial x} \int_{t_1}^{t_2} \theta W_x\, dt - \theta W_x \frac{\partial t_2}{\partial x} + \theta W_x \frac{\partial t_1}{\partial x}$$

Since t_1 and t_2 are independent of x, the last two terms above are both 0. Using the definition (4), the first term on the R.H.S. may be simplified

$$\frac{\partial}{\partial x} \int_{t_1}^{t_2} \theta W_x\, dt = T\, \frac{\partial(\overline{\theta W_x})}{\partial x} ,$$

where the overbar indicates the time average and $T \equiv t_2 - t_1$.
Using Reynolds law of averaging

$$\overline{\theta W_x} \equiv \bar{\theta}\, \overline{W}_x + \overline{\theta' W_x'} ,$$

where θ' and W_x' represent the perturbations from the mean values.
The second term in (5) may now be written:

$$\int_{t_1}^{t_2} \frac{\partial(\theta W_x)}{\partial x}\, dt = T\, \frac{\partial(\bar{\theta}\, \overline{W}_x)}{\partial x} + T\, \frac{\partial(\overline{\theta' W_x'})}{\partial x}$$

The flux term, $\overline{\theta' W'}_x$, may be approximated using the gradient transfer hypothesis. It is assumed that the flux of pollutant is proportional to the concentration gradient. This hypothesis has been discredited when applied to parameters which affect the ambient flow, such as momentum or temperature. However, there is no reason to believe that it is not a good approximation for a passive contaminant. The utility of the hypothesis has been demonstrated for a concentration distribution due to an instantaneous point source (Hutcheson 1979).

$$\therefore \quad \overline{\theta' W_x'} = -\,\varepsilon_x \frac{\partial \theta}{\partial x}$$

where ε_x is the dispersion coefficient in the x direction.

$$\therefore \int_{t_1}^{t_2} \frac{\partial(\theta W_x)}{\partial x}\,dt = T\,\frac{\partial(\bar{\theta}\;\overline{W}_x)}{\partial x} - T\,\frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial \bar{\theta}}{\partial x}\right) \qquad (6)$$

Similarly, the third and forth terms in (5) may be evaluated.

$$\int_{t_1}^{t_2} \frac{\partial(\theta W_y)}{\partial y}\,dt = T\,\frac{\partial(\bar{\theta}\;\overline{W}_y)}{\partial y} - T\,\frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial \bar{\theta}}{\partial y}\right) \qquad (7)$$

$$\int_{t_1}^{t_2} \frac{\partial(\theta W_z)}{\partial z}\,dt = T\,\frac{\partial(\bar{\theta}\;\overline{W}_z)}{\partial z} - T\,\frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial \bar{\theta}}{\partial z}\right) \qquad (8)$$

Substitution of (6), (7), and (8) into (5) yields

$$\frac{\partial(\bar{\theta}\overline{W}_x)}{\partial x} + \frac{\partial(\bar{\theta}\overline{W}_y)}{\partial y} + \frac{\partial(\bar{\theta}\overline{W}_z)}{\partial z} - \left\{\frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial \bar{\theta}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial \bar{\theta}}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial \bar{\theta}}{\partial z}\right)\right\} = 0 \qquad (9)$$

Equation (9) will be transformed from the Cartesian to the Natural coordinate system for several reasons. Most importantly for the wasteload allocation process, a coordinate transformation allows a solution to an appropriate dispersion equation involving only the four independent variables C, C_B, Q_α and Q_e with the minimum number of assumptions. Furthermore, as shown by Yotsukura and Sayre (1976), the Natural coordinate system allows the concentration distribution to more closely resemble a normal distribution. This is important since, as we shall see, the analytical solution obtained is in fact a normal distribution (or, more accurately, half - normal). The only turbulent diffusion terms included in (9) are those involving the principal (scalar) diffusivities. However, this is only appropriate if the principal axes of the diffusion tensor are in alignment with the coordinate directions. This can only be accomplished in a Natural coordinate system.

In the Natural system, the horizontal coordinates are x, the stream thalweg and z, perpendicular to x. The vertical coordinate remains the same as in the Cartesian system. In order to convert from Cartesian coordinates, define

$$x \equiv m_x \bar{x}, \quad y \equiv \bar{y}, \quad z \equiv m_z \bar{z} .$$

The m_x and m_z are introduced to correct for differences between distances along a curved coordinate surface and those measured along the respective straight Cartesian axes.

If

$$\bar{w}_x = \frac{d\bar{x}}{dt}, \text{ and } \quad u_x \equiv \frac{dx}{dt},$$

$$u_x = m_x \frac{d\bar{x}}{dt} = m_x \bar{w}_x .$$

Similarly

$$u_z = m_z \bar{w}_z .$$

$$\therefore \quad \bar{w}_x = \frac{u_x}{m_x}, \qquad \bar{w}_z = \frac{u_z}{m_z} \tag{10}$$

Generally both m_x and m_z vary with x and z. An approximate relationship for m_z in a channel where the longitudinal coordinate (x) surfaces are approximately evenly spaced is given by (Yotsukura and Sayre 1976)

$$m_z \simeq \frac{L_t}{L_z}$$

where L_t is the channel width measured along a transverse coordinate surface and L_z is the width along the z axis. Except in the middle of a sharp bend, $L_t \cong L_z$. Therefore,

$$m_z \simeq 1 .$$

For wasteload allocation purposes we are interested in one concentration only; the maximum concentration outside the mixing zone. For this reason, m_x will be considered an average between the source and the maximum concentration and is therefore independent of x.

Defining $u_y \equiv \bar{W}_y$ and $c \equiv \bar{\theta}$, using (10) and m_x in (9) yields

$$\frac{1}{m_x}\frac{\partial(cu_x)}{\partial x} + \frac{\partial(cu_y)}{\partial y} + \frac{\partial(cu_z)}{\partial z} = \frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon_y \frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right) \tag{11}$$

In order to simplify the diffusion equation further, it is assumed that the pollutant is uniformly mixed in the vertical. Yotsukura and Sayre (1976) estimated that uniform vertical mixing usually occurs within 50 to 100 times the depth at the discharge point. Since the discharge is typically at the bank, uniform vertical mixing will usually have occurred at the point of maximum concentration in the region where water quality standards apply. Therefore, it may be assumed that

$$\frac{\partial c}{\partial y} = 0.$$

Under this assumption, the middle term on the R.H.S. of (11) may be neglected. Furthermore, it must be assumed that conservative parameters are neutrally bouyant. This is a necessary assumption to obtain an analytical solution, even though some conservative parameters are definitely negatively bouyant. The assumption of neutral buoyancy sometimes makes the allocation conservative, and allows

$$\frac{\partial u_y}{\partial y} = 0.$$

The middle term L.H.S. of (11) may be expanded

$$\frac{\partial (cu_y)}{\partial y} = c\,\frac{\partial u_y}{\partial y} + u_y\,\frac{\partial c}{\partial y} = 0,$$

so, the middle terms on both sides of (11) can be eliminated to obtain

$$\frac{1}{m_x}\frac{\partial (cu_x)}{\partial x} + \frac{\partial (cu_z)}{\partial z} = \frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right) \qquad (12)$$

Vertical integration must be performed to obtain a depth averaged equation, since u_x and u_z are not uniform in the vertical

$$\begin{aligned} \therefore\ \frac{1}{m_x}\int_{Y_b}^{Y_s} \frac{\partial (cu_x)}{\partial x}\,dy + \int_{Y_b}^{Y_s} \frac{\partial (cu_z)}{\partial z}\,dy \\ = \int_{Y_b}^{Y_s} \frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial c}{\partial x}\right) dy + \int_{Y_b}^{Y_s} \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right) dy \end{aligned} \qquad (13)$$

where Y_b and Y_s are the elevations of the stream bottom and the surface, respectively.

Leibnitz's rule may be used on the first term in (13) to obtain

$$\int_{Y_b}^{Y_s} \frac{\partial(cu_x)}{\partial x}\,dy = \frac{\partial}{\partial x}\int_{Y_b}^{Y_s} cu_x\,dy - cu_x\frac{\partial Y_s}{\partial x} + cu_x\frac{\partial Y_b}{\partial x}$$

Since Y_s and Y_b change slowly on most streams, the last two terms may be neglected over the distances involved in wasteload allocation.

Using the averaging definition (4)

$$\frac{\partial}{\partial x}\left(\int_{Y_b}^{Y_s} cu_x\,dy\right) = \frac{\partial}{\partial x}\left(h\overline{cu}_x\right)$$

where the overbar now indicates a vertical average, and $h = Y_s - Y_b$.

Using Reynold averaging,

$$\overline{cu}_x = c\bar{u}_x + \overline{c'u'_x}$$

Since c is assumed uniformly distributed in the vertical, no overbar is necessary to denote this average.

The gradient transfer hypothesis may be used to obtain

$$\overline{c'u'_x} = -E_x\frac{\partial c}{\partial x},$$

where E_x is a dispersion coefficient resulting from vertical (as opposed to time) average.

Therefore, the first term in (13) becomes

$$\frac{1}{\overline{m}_x}\int_{Y_b}^{Y_s} \frac{\partial(cu_x)}{\partial x}\,dy = \frac{1}{\overline{m}_x}\frac{\partial(hc\bar{u}_x)}{\partial x} - \frac{1}{\overline{m}_x}\frac{\partial}{\partial x}\left(hE_x\frac{\partial c}{\partial x}\right) \qquad (14)$$

Leibnitz's rule may be used on the second term in (12) to obtain

$$\int_{Y_b}^{Y_s} \frac{\partial(cu_z)}{\partial z}\,dy = \frac{\partial}{\partial z}\int_{Y_b}^{Y_s} cu_z\,dy - cu_z\frac{\partial Y_s}{\partial z} + cu_z\frac{\partial Y_b}{\partial z}$$

Again, the last two terms may be neglected, particularly in midwestern streams, which tend to have rather flat gradients.

Using the averaging definition (4) again,

$$\frac{\partial}{\partial z}\int_{Y_b}^{Y_s} cu_z \, dy = \frac{\partial(h\bar{c}\bar{u}_z)}{\partial z}$$

Reynolds averaging and the gradient transfer hypothesis yields

$$\frac{\partial(h\bar{c}\bar{u}_z)}{\partial z} = \frac{\partial(h\bar{c}_z)}{\partial z} - \frac{\partial}{\partial z}\left(hE_z\frac{\partial c}{\partial z}\right)$$

Therefore, the second term in (13) may be written

$$\int_{Y_b}^{Y_s} \frac{\partial(cu_z)}{\partial z}\, dy = \frac{\partial(hc\bar{u}_z)}{\partial z} - \frac{\partial}{\partial z}\left(hE_z\frac{\partial c}{\partial z}\right)$$

In the Natural coordinate system, the mean flow is in the x direction. Therefore, $u_z = 0$, and

$$\int_{Y_b}^{Y_s} \frac{\partial(cu_z)}{\partial z}\, dy = -\frac{\partial}{\partial z}\left(hE_z\frac{\partial c}{\partial z}\right) \tag{15}$$

Equation (15) implies that there are no cross currents in the stream after a vertical average is taken. Except in the vicinity of large eddies, this is a reasonable assumption. Even in the vicinity of river bends, where spiral secondary currents are formed by the change in flow direction, so that there is a current towards the outside of the bend in the upper portion of the flow, and a current towards the inside of the bend in the lower portion, the net transverse flow is still zero. Of course, this phenomena will tend to increase dispersion in the transverse direction, and must be accounted for by an increase in E_z in the vicinity of river bends. Since the dispersion coefficient does not appear in the wasteload allocation, changes in E_z do not have to be accounted for explicitly here.

The first term on the R.H.S. of (13) may be evaluated by assuming that the integral is independent of x.

$$\int_{Y_b}^{Y_s} \frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial c}{\partial x}\right) dy = \frac{\partial}{\partial x}\left(\varepsilon_x\frac{\partial c}{\partial x}\int_{Y_b}^{Y_s} dy\right)$$

since

$$\int_{Y_b}^{Y_s} dy = Y_s - Y_b = h,$$

$$\int_{Y_b}^{Y_s} \frac{\partial}{\partial x}\left(\varepsilon_x \frac{\partial c}{\partial x}\right) dy = \frac{\partial}{\partial x}\left(h\varepsilon_x \frac{\partial c}{\partial x}\right) \qquad (16)$$

The last term in (12) may be similarly evaluated

$$\int_{Y_b}^{Y_s} \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial c}{\partial z}\right) dy = \frac{\partial}{\partial z}\left(h\varepsilon_z \frac{\partial c}{\partial z}\right) \qquad (17)$$

The assumption that the stream surface elevation is independent of the horizontal coordinates is a good one for low gradient streams. The validity of the assumption of independence of stream bottom elevation depends on the morphology of the stream. For wasteload allocation purposes, these assumptions will generally be valid, since discharges are not prevalent on high gradient steams.

Defining $u \equiv \overline{u}_x$ and substituting (14), (15), (16), and (17) into (13) yields

$$\frac{1}{m_x}\frac{\partial (hcu)}{\partial x} = \frac{1}{m_x}\frac{\partial}{\partial x}\left(hE_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial x}\left(h\varepsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial z}\left(hE_z \frac{\partial c}{\partial z}\right) + \frac{\partial}{\partial z}\left(h\varepsilon_z \frac{\partial c}{\partial z}\right) \qquad (18)$$

Longitudinal dispersion has very little influence on steady state mixing in the Natural coordinate system, since the longitudinal coordinate surface will follow the main flow direction (Yotsukura and Sayre 1976). Even though longitudinal dispersion is the dominate dispersion mechanism for instantaneous discharges, longitudinal concentration gradients are so small under steady state conditions they render this mechanism ineffective. This is another reason why the choice of the Natural coordinate system is a good one. Since longitudinal dispersion may be neglected, the terms in (18) involving ε_x and E_x can be eliminated for the purpose of wasteload allocation.

The two dispersion terms arising from vertical, E_z, and temporal, ε_z, averaging may be combined by defining

$$K \equiv E_z + \varepsilon_z$$

so that

$$\frac{\partial(hcu)}{\partial x} = m_x \frac{\partial}{\partial z}\left(hK\frac{\partial c}{\partial z}\right) \qquad (19)$$

The transverse (z) coordinate may be changed to a cumulative discharge (q) coordinate through the use of the chain rule. The cumulative discharge coordinate may be expressed as

$$q \equiv \int hudz, \text{ so that } \frac{\partial q}{\partial z} = hu \qquad (20)$$

The cumulative discharge, q, is the flow in the region between the origin (z = q = 0) and a point z. Thus, if the origin is located at one bank, at the other bank (q) equals the entire stream flow. A more detailed explanation of the Natural coordinate system is given by Yotsukura and Sayre (1976).

Applying the chain rule to the R.H.S. of (19),

$$\frac{\partial c}{\partial z} = \frac{\partial c}{\partial q}\;\frac{\partial q}{\partial z}$$

Using (20)

$$\frac{\partial c}{\partial z} = hu\;\frac{\partial c}{\partial q}$$

$$\therefore\quad \frac{\partial}{\partial z}\left(hK\frac{\partial c}{\partial z}\right) = \frac{\partial}{\partial z}\left(h^2\,Ku\frac{\partial c}{\partial q}\right)$$

Applying the chain rule again and using (20)

$$\frac{\partial}{\partial z}\left(hK\frac{\partial c}{\partial z}\right) = hu\frac{\partial}{\partial q}\left(h^2\,Ku\frac{\partial c}{\partial q}\right) \qquad (21)$$

Using the product rule, the first term in (19) may be written

$$\frac{\partial\,(hcu)}{\partial x} = hu\frac{\partial c}{\partial x} + c\frac{\partial hu}{\partial x}$$

In the Natural coordinate system, the product hu varies slowly in the x direction. When the depth decreases, speed increases, and vice versa. This is not necessarily the case in the Cartesian system, because the x axis is not located along the stream thalweg. The product of h and u varies slowly with

x. In areas where a stream is shallow, it tends to be fast. Where it is deep, it tends to be slow. Therefore, the last term may be neglected, so

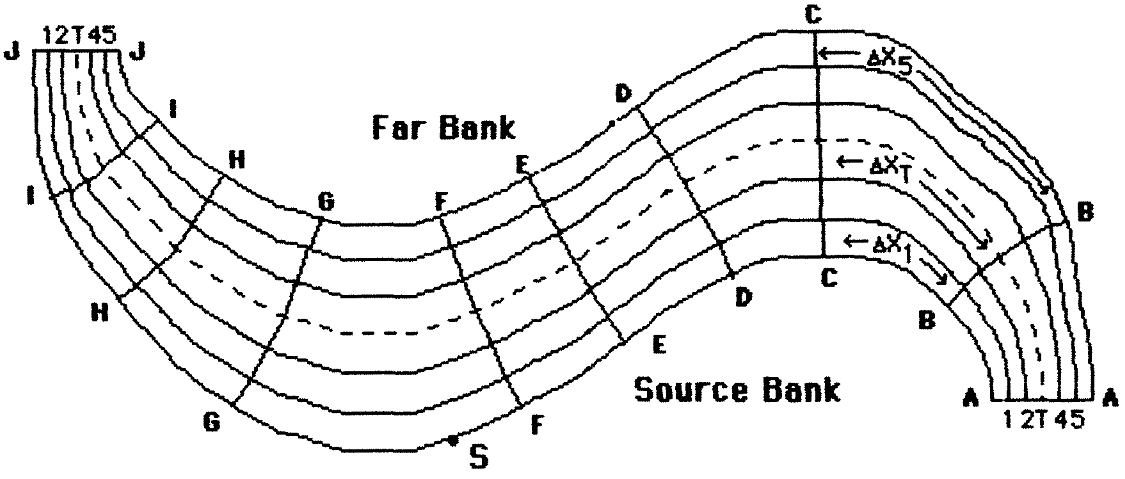
$$\frac{\partial(hcu)}{\partial x} = hu\frac{\partial c}{\partial x} .$$

Substituting this expression and (21) into (19), we obtain

$$\frac{\partial c}{\partial x} = \bar{m}_x \frac{\partial}{\partial q}\left(h^2 Ku \frac{\partial c}{\partial q}\right) \tag{22}$$

This equation is similar to that introduced by Yotsukura and Sayre (1976). Many more assumptions than necessary were made here to obtain this equation. However, these assumptions will later be shown to be necessary to obtain an analytical solution. Changing from the distance to the cumulative discharge coordinate is a crucial step in obtaining a solution which requires only the same parameters used in mass balance with a minimum number of assumptions.

To compute concentration from (22) using only the parameters required by mass balance, $\bar{m}_x$ must be evaluated. Since the x axis of the Natural coordinate system lies along the thalweg, m_x is always 1 on the thalweg. The $\bar{m}_x$ is the fraction of the distance along the thalweg between transverse coordinate surfaces for longitudinal coordinate surfaces other than the thalweg. In figure 2, Δx_1 is a small fraction of Δx_T, and Δx_5 is much larger than Δx_T. However, when the source is located on surface D, $m_x \approx 1$ for any longitudinal coordinate surface. Since $\bar{m}_x$ is an average, $\bar{m}_x = 1$ in Figure 2 everywhere on transverse coordinate J, if the source is at transverse coordinate A. Therefore, $\bar{m}_x \approx 1$, even when river bends occur, if the maximum concentration on the mixing zone boundary occurs far enough downstream from the source. Therefore, for wasteload allocation purposes, $\bar{m}_x = 1$ is generally a reasonable assumption, even though $m_x = 1$ is not, except along the thalweg.



PLAN VIEW

T (dotted line) = stream thalweg
1, 2, 4, 5 = longitudinal coordinate surfaces
A, B, C, ... I, J = transverse coordinate surfaces
S = source of pollution
$\Delta X_1 = \bar{m}_{X_1} \, \Delta X_T \qquad \bar{m}_{X_1} < 1$
$\Delta X_5 = m_{X_5} \, \Delta X_T \qquad m_{X_5} > 1$
longitudinal and transverse coordinate are always ⊥.

Figure 2. The coordinate surfaces on the Natural coordinate system.

To obtain an analytical solution to (22), further simplification is required. The dispersion coefficient may be assumed independent of q. Usually K is assumed constant, and while this is certainly not true in all cases, assuming K independent of q is valid for wasteload allocation purposes, because K dictates the spread of pollutant across the stream.

It cannot be assumed that h^2u is independent of q. Near the banks, h and u both change rapidly. Expanding the R.H.S. of (22),

$$\frac{\partial}{\partial q}\left(h^2 uK \frac{\partial c}{\partial q}\right) = K \frac{\partial c}{\partial q} \frac{\partial (h^2 u)}{\partial q} + Kh^2 u \frac{\partial^2 c}{\partial q^2} .$$

Near the banks, where $\partial(h^2u)/\partial q$ is large, $\partial c/\partial q$ is small under the wasteload allocation conditions. Near the center of the stream, $\partial c/\partial q$ is large under wasteload allocation conditions, but $\partial(h^2u)/\partial q$ is small. Thus, the first term on the R.H.S. is never large, and may be neglected. Therefore, equation (22) may be written:

$$\frac{\partial c}{\partial x} = h^2 K u \frac{\partial^2 c}{\partial q^2} \tag{23}$$

Equation (23) may be solved analytically to obtain the concentration at a given point, if the proper boundary conditions are employed. Rewriting (23),

$$\frac{\partial^2 c}{\partial q^2} - \frac{1}{h^2 K u}\frac{\partial c}{\partial x} = 0 \tag{24}$$

The appropriate boundary conditions are (Yotsukura and Sayre 1976):

$$\begin{aligned} &c \rightarrow 0 \quad \text{as} \quad q \rightarrow \infty \\ &c \rightarrow 0 \quad \text{as} \quad x \rightarrow 0, \quad q \neq 0 \\ &S = \int_{-\infty}^{\infty} c\,dq \end{aligned} \tag{25}$$

where S is the source strength, or wasteload, which may be defined under steady state conditions as the product of the effluent flow and concentration ($S \equiv C_e Q_e$). This assumes that pollutant is mixed uniformly throughout the effluent. These boundary conditions constrain the concentration to be zero in the stream at positions transverse to the source, and require that mass be conserved (i.e. there are no sources or sinks of pollution other than S)

The Laplace transform of (24)

$$L\left\{\frac{\partial^2 c}{\partial q^2}\right\} - L\left\{\frac{1}{h^2 K u}\frac{\partial c}{\partial x}\right\} = 0,$$

may be obtained using

$$f(p) = L\{F(x)\} = \int_0^{\infty} e^{-px} F(x)\,dx$$

where $f(p)$ is the Laplace transform of $F(x)$, and $F(x)$ is a real function of x; $F(x) = 0, x < 0$.

$$\therefore \int_0^\infty e^{-px} \frac{\partial^2 c}{\partial q^2}\, dx - \frac{1}{h^2 K u} \int_0^\infty e^{-px} \frac{\partial c}{\partial x}\, dx = 0 \tag{26}$$

where K and the product h^2u are assumed independent of x. This is an unacceptable assumption near the source, when the plume in its initial chaotic mixing phase. Dispersion of a pollutant from a steady state point source is dependent upon both the characteristics of the prevalent turbulent eddies and the size of the pollutant plume. Very near the source the relationship between the concentration variance and distance downstream is highly nonlinear (Hutcheson 1981). Further from the source this relationship tends to become more linear, implying that in this region K approaches a constant value. It is assumed that the dependence of K upon x has ceased at the distance downstream where the concentration on a mixing zone boundary is a maximum.

The second term in (26) may be evaluated using $\int u\,dv = uv - \int v\,du$ (Handbook of Chemistry and Physics, 42nd Edition).

$$\therefore \int_0^\infty e^{-px} \frac{\partial c}{\partial x}\, dx = e^{-px} c \Big|_0^\infty + p \int_0^\infty e^{-px} c\, dx$$

$$= -c_{x=0} + p \int_0^\infty e^{-px} c\, dx$$

Since, from (25), $C_{x=0} = 0$, the second term in (26) becomes

$$\frac{1}{h^2 K u} \int_0^\infty e^{-px} \frac{\partial c}{\partial x}\, dx = \frac{p}{h^2 K u} \bar{c} \tag{27}$$

where

$$\bar{c} \equiv \int_0^\infty e^{-px}\, c\, dx \tag{28}$$

$\bar{c}$ is the Laplace transform of c. Since e^{-px} is independent of q, the first term in (26) may be written

$$\int_0^\infty e^{-px} \frac{\partial^2 c}{\partial q^2}\, dx = \frac{\partial^2}{\partial q^2} \int_0^\infty e^{-px} c\, dx = \frac{\partial^2 \bar{c}}{\partial q^2} \quad \text{using (28).} \tag{29}$$

Substitution of (27) and (29) into (26) yields

$$\frac{\partial^2 \bar{c}}{\partial q^2} - \frac{p}{h^2 K u} \bar{c} = 0 \tag{30}$$

Since c is independent of x (only dependent on q) (30) is an ordinary differential equation, and may be written as

$$\frac{d^2\bar{c}}{dq^2} - \frac{p}{h^2Ku}\bar{c} = 0 \qquad (31)$$

called the subsidiary equation.

To facilitate the solution to the ordinary differential equation, define

$$a^2 = \frac{p}{h^2Ku} \qquad (32)$$

Substitution into (31) yields

$$\frac{d^2\bar{c}}{dq^2} - a^2\bar{c} = 0 \qquad (33)$$

The auxiliary equation for (33) is

$$m^2 - a^2 = 0 \qquad \therefore \quad m = a, \quad m = -a,$$

so the solution to (33) may be expressed as

$$\bar{c} = C_1e^{-aq} + C_2e^{aq} \qquad (34)$$

From (28) we see that if $c = 0, \bar{c} = 0$. Therefore, B.C. (25) may be written as $\bar{c} \to 0$ as $q \to \infty$, so (34) becomes $0 = C_2e^{aq}, q \to \infty$. This can only be true if $C_2 = 0$. Substitution in (34) yields

$$\bar{c} = C_1e^{-aq} \qquad (35)$$

To determine C_1, the boundary condition (25) must be used again. Due to the symmetry of the distribution in the q direction (half of the mass on either side of the axis) (25) may be written

$$\int_0^\infty c\,dq = \frac{S}{2} \qquad (36)$$

Taking the Laplace transform of (36)

$$L\left\{\int_0^\infty c\,dq\right\} = L\left\{\frac{S}{2}\right\}$$

$$\int_0^\infty e^{-px}\int_0^\infty c\,dq\,dx = \frac{1}{2}\int_0^\infty e^{-px}\,S\,dx$$

$$\int_0^\infty\int_0^\infty e^{-px}\,c\,dx\,dq = \frac{S}{2}\int_0^\infty e^{-px}\,dx,$$

since the source strength is constant.

Using definition (28)

$$\int_0^\infty \bar{c}\,dq = \frac{S}{2p}\,e^{-px}\Big|_0^\infty = -\frac{S}{2p}\,[0-1]$$

$$\therefore \int_0^\infty \bar{c}\,dq = \frac{S}{2p} \tag{37}$$

Substituting from (35) for c, (37) becomes

$$C_1\int_0^\infty e^{-aq}\,dq = \frac{S}{2p} \tag{38}$$

operating on the L.H.S.,

$$C_1\int_0^\infty e^{-aq}\,dq = -\frac{C_1}{a}\,e^{-aq}\Big|_0^\infty = -\frac{C_1}{a}\,[0-1] = \frac{C_1}{a}$$

Substituting into (38) yields

$$\frac{C_1}{a} = \frac{S}{2p} \quad \text{or} \quad C_1 = \frac{aS}{2p} \tag{39}$$

Substitution of (39) into (35) yields

$$\bar{c} = \frac{aS}{2p}\,e^{-aq}$$

Substitution for a from (32) yields

$$\bar{c} = \frac{S}{2h\sqrt{ku}}\left(\frac{1}{\sqrt{p}}\,e^{-k\sqrt{p}}\right) \tag{40}$$

where $k \equiv \dfrac{q}{h\sqrt{Ku}}$.

To obtain an expression for the concentration, the inverse Laplace transform of (40) must be performed. Remember that the inverse Laplace transform of $\bar{c}$ is c, from (28).

When the Laplace transform is $\frac{1}{\sqrt{p}} e^{-k\sqrt{p}}$, then the inverse Laplace transformation is $\frac{1}{\sqrt{\pi x}} \exp\left(-\frac{k^2}{4x}\right)$ (Handbook of Chemistry and Physics, 42nd edition). Therefore, from (40), and substituting for k

$$c = \frac{S}{2h\,(Ku\pi x)^{1/2}} \exp\left(-\frac{q^2}{4h^2Kux}\right) \tag{41}$$

This solution to the dispersion equation (24) is not particularly useful, because it involves the dispersion coefficient K, which is not well known, Furthermore, it requires knowledge of h and u, which is difficult to obtain, and is not required for the mass balance approach.

Taking advantage of an observation by Yotsukura and Sayre (1976) that, in the Natural coordinate system (but not in a Cartesian system) the concentration distribution in the q direction is approximately normal, the troublesome K, h and u may be eliminated. When a variable is normally distributed in the q direction, the probability density, n (q) is given by

$$n(q) \equiv \frac{1}{\sqrt{2\pi}\,\sigma} e^{-q^2/2\sigma^2}, \tag{42}$$

where the mean of the distribution is located at q = 0, and sigma is the standard deviation of the normal distribution. The equation is normalized, hence the total area under the curve is unity. Therefore,

$$\int_{-\infty}^{\infty} n(q)\,dq = 1 \tag{43}$$

From (25),

$$\int_{-\infty}^{\infty} \frac{c}{S}\,dq = 1, \text{ since S is independent of q.}$$

Comparing this with (43), and using the assumption that the concentration distribution is normal,

$$\frac{c}{S} = n(q), \text{ and equation (41) may be written:}$$

$$\frac{c}{S} = n(q) = \frac{1}{2h\,(Ku\pi x)^{1/2}} \exp\left(-\frac{q^2}{4h^2Kux}\right) \qquad (44)$$

Comparing (42) and (44),

$$K \equiv \frac{\sigma^2}{2h^2ux} \qquad (45)$$

There is no evidence that (45) is a valid expression for the dispersion coefficient. However, assuming it is valid, (45) clearly shows what was earlier stated: in regions where σ^2 does not change linearly with distance from the source, K is not constant, and one of the assumptions which this derivation rests upon is not valid. The assumption that K is independent of x is only valid when σ^2 increases linearly with x.

Substituting (45) into (41) yields

$$c = \frac{S}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{q^2}{2\sigma^2}\right) \qquad (46)$$

This equation has eliminated the troublesome h, u and K, but introduced a new variable . However, σ can be determined analytically for the special case needed in wasteload allocation with no knowledge of the dispersion characteristics of the pollutant plume. This will be accomplished in the next chapter.

Although the expression for the dispersion coefficient (45) cannot be shown to be correct, it can be shown that (46) satisfies the differential equation (24) and the boundary conditions (25).

A comparison of (46) and (25) shows that (46) satisfies the boundary conditions. As $q \to \infty$, (46) forces c to 0. As $x \to 0$, $\sigma \to 0$, $\exp(-q^2/2\sigma^2) \to 0$ rapidly, so $c \to 0$. Mass conservation may be investigated by substituting (46) into (25) to obtain

$$S = S\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-q^2/2\sigma^2}\, dq = S \text{, using (42) and (43).}$$

It may be shown that not only the boundary conditions, but also the differential equation (24) is satisfied by (46). In order to do this, (45) must be differentiated to obtain

$$d\sigma^2 = 2Kh^2u\,dx \qquad \text{or}$$

$$K = \frac{1}{2h^2 u}\,\frac{d\sigma^2}{dx}$$

This is only valid in the region where K is constant. Since σ is only dependent on x, this may be written

$$K = \frac{1}{2h^2 u}\,\frac{\partial\sigma^2}{\partial x}\ .$$

Differentiating (46);

$$\frac{\partial c}{\partial x} = \frac{S}{\sqrt{2\pi}\,\sigma^2}\left\{\frac{q^2}{\sigma^2} - 1\right\}\exp\left(-\frac{q^2}{2\sigma^2}\right)\frac{\partial\sigma}{\partial x}$$

and

$$\frac{\partial^2 c}{\partial q^2} = \frac{S}{\sqrt{2\pi}\,\sigma^3}\left\{\frac{q^2}{\sigma^2} - 1\right\}\exp\left(-\frac{q^2}{2\sigma^2}\right).$$

Substitution for K, $\partial c/\partial x$ and $\partial^2 c/\partial q^2$ in (24) shows that the dispersion equation is indeed satisfied. Therefore, although the validity of some of the assumptions involved in obtaining the solution (46) may be questionable, at least it satisfies a valid dispersion equation and the boundary conditions.

When the source is located on the stream bank, rather than near the center of an infinitely wide stream, perfect reflection from the source bank may be assumed. This assumption results in a half normal distribution, so that the concentration at any point is doubled. From (46)

$$c = \frac{2S}{\sqrt{2\pi}\,\sigma}\exp\left(-\frac{q^2}{2\sigma^2}\right) \tag{47}$$

The background concentration, C_B, may be added directly to the concentration caused by the effluent discharge, due to the principal of superposition. Since C_B is assumed constant throughout the stream reach being considered in the wasteload allocation, this holds true no matter what coordinate system is used. Therefore, from (47)

$$c = \frac{\sqrt{2}\,S}{\sqrt{\pi}\,\sigma}\exp\left(-\frac{q^2}{2\sigma^2}\right) + C_B \tag{48}$$

The larger the background concentration, the lower the dilution capacity of the stream. Therefore, taking background concentration into account is very important in the wasteload allocation.

As the pollutant plume spreads across the stream, reflection from the far bank will occur. Equation (48) does not account for this. Accounting for refection from the far bank is not necessary so long as the edge of the mixing zone is near the injection bank, because increased concentration due to reflection at the point of maximum concentration on the mixing zone boundary is negligible in this case.

All states which currently define a mixing zone allow less than one-half the flow volume for it (EPA 1980). Therefore, reflection from the far bank need not be accounted for in the wasteload allocation.

Equation (48) is the analytical solution to the dispersion equation which will form the heart of the wasteload allocation. Although many assumptions were required to obtain this solution, each assumption was discussed and justified. Because of this, we can expect that the concentration prediction obtained from (48) will be adequate for wasteload allocation purposes in many instances.

Chapter 4

Development of the Wasteload Allocation

The wasteload allocation is being developed to ensure that the maximum concentration on the mixing zone boundary is always less than or equal to the numerical criterion in the water quality standards. The mixing zone and zone of passage are shown in Figure 3. Water quality standards cannot be exceeded in the zone of passage. Note the mixing zone boundary, source location and direction of flow.

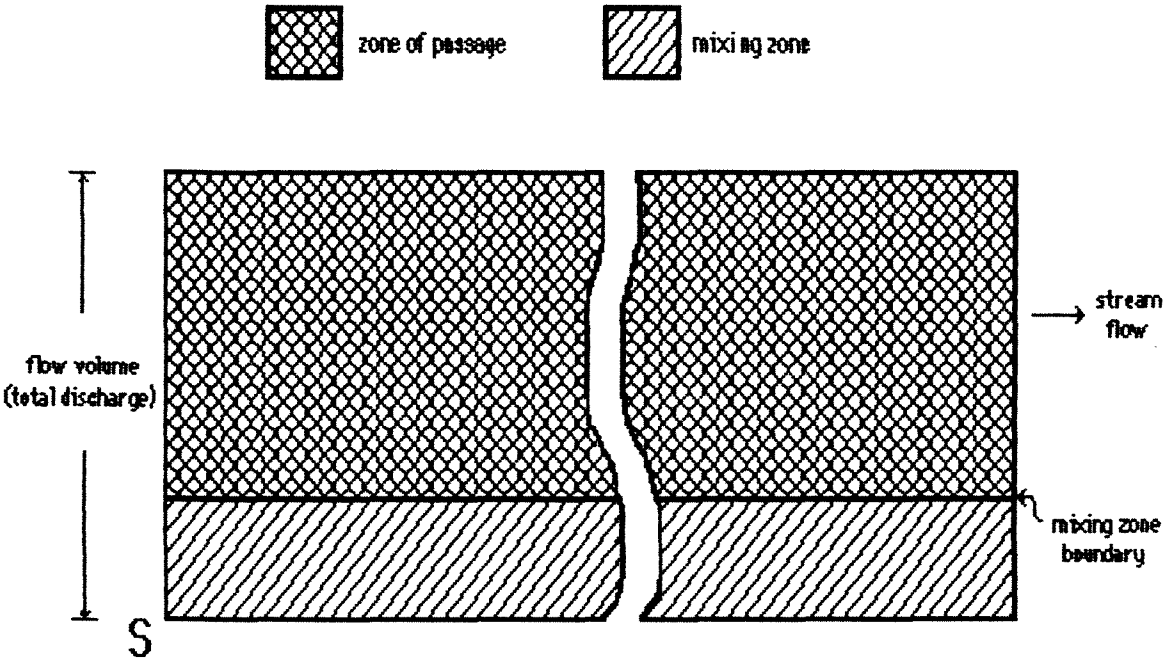


Figure 3. Portion of receiving stream in which water quality standards are not valid for an isolated discharge.

The mixing zone boundary may be assumed infinitely long for wasteload allocation purposes, since the only point of importance on the boundary is where the concentration is a maximum. The portion of the flow, Q_M, reserved for a mixing zone varies from state to state. The cumulative discharge at the mixing zone boundary equals this portion of the flow. Therefore;

$$q = Q_M \tag{49}$$

for wasteload allocation purposes.

In order to insure that standards are met everywhere outside this mixing zone, the permit limit must constrain the effluent so that the concentration at the point on the mixing zone boundary where the concentration is a maximum does not exceed the applicable numerical criterion. Gowda (1984) determined the distance to the maximum concentration on the mixing zone boundary by differentiating a dispersion equation and setting the derivative equal to zero.

We may use a similar approach to determine σ when the concentration on the mixing zone boundary is a maximum. Differentiating (48) ω/r x,

$$\frac{\partial c}{\partial x} = 0 = \frac{\sqrt{2}\,S}{\sqrt{\pi}} \frac{\partial}{\partial x} \left[\frac{e^{-q^2/2\sigma^2}}{\sigma} \right].$$

Since σ is the only variable which is dependent upon x,

$$\frac{\partial}{\partial x} \left[\frac{e^{-q^2/2\sigma^2}}{\sigma} \right] = 0, \quad \text{or} \quad \sigma = q .$$

This happy coincidence allows development of a wasteload allocation using the same input parameters as required by the mass balance allocation. Since the mixing zone boundary is located at Q_M, using (49),

$$\sigma = Q_M, \tag{50}$$

when the concentration on the mixing zone boundary is a maximum.

Substitution of (50) into (48) yields

$$c - C_B = \frac{\sqrt{2}\,S}{\sqrt{\pi}\,Q_M} e^{-\frac{1}{2}} .$$

Performing the appropriate mathematical manipulations and using the definition of S in (25),

$$C_e = 2.066 \frac{C' Q_M}{Q_e} , \tag{51}$$

where

$$C' = c - C_B .$$

Equation (51) yields an effluent concentration which, when discharged to the receiving stream, will not violate water quality standards if $c \equiv C$ (defined as the numerical criterion). If the background concentration is greater than the standard, $C' = C - C_B$ is negative, so the effluent concentration allowed by (51) becomes negative. Rational water quality management must allow an allocation where the concentration in the discharge is at least as great as the water quality standard. Therefore, for wasteload allocation purposes, (51) will always be combined with the requirement that $C_e \geq C$.

Under steady state conditions, the wasteload may be obtained using C_e and Q_e in the definition of S in (25). Since Q_M is some fraction of the sum of Q_u and Q_e, (51) satisfies our requirements for a wasteload allocation; namely that only C, C_B, Q_e and Q_u are required to obtain C_e, and that the concentration be less than or equal the numerical criteria in the water quality standards throughout the region in which the standards are applicable.

The distance from the source to the maximum concentration on the mixing zone boundary is highly variable. In some receiving streams the plume will spread very rapidly, so that the maximum concentration on the mixing zone boundary will be relatively close to the source. In other streams the plume will spread slowly, so this maximum concentration will be further downstream. As the morphology of the receiving stream changes, the distance downstream to maximum concentration will vary greatly within the stream, depending on source location.

The nature of the discharge itself affects the rate of plume spread, hence location of maximum concentration. The greater the momentum of the discharge relative to the receiving stream, the faster the plume will spread, and vice-versa. Fortunately (51) does not require any information about the location of the maximum concentration on the mixing zone boundary. Therefore, for wasteload allocation purposes, the location of the maximum concentration is unimportant.

It is important to reiterate that no empiricism has been incorporated into this wasteload allocation. For example, no estimate of the dispersion coefficient is required. In those situations where the many assumptions required to obtain the wasteload allocation are valid, (51) will yield a wasteload allocation which will protect the numerical criteria in the standards throughout the region they apply.

The wasteload allocation may easily be extended to multiple dischargers, using the principal of superposition. However, the dispersion coefficient mean depth and velocity must be determined through an intensive instream survey. Equation (51) may also be easily converted for use with certain non-conservative substances, such as chlorine or ammonia. However, the dispersion coefficient, mean depth and velocity, as well as the decay characteristics of the substance, will be required.

Although provisions for multiple dischargers and certain non-conservative substances may be added to the wasteload allocation to make it more versatile, they would require the use of additional input parameters. One of the constraints placed on the allocation developed here is that only the parameters used in the mass balance allocation be required, so these refinements are beyond the scope of this work.

The wasteload allocation derivation presented here is not the simplest one possible. The simplest derivation assumes a normal distribution in Cartesian coordinates with a constant, uniform flow. The standard dispersion equation for a Gaussian distribution could then be converted to equation (51). This approach is not used because the assumptions required are unnecessarily stringent. A uniform channel must be assumed, rather than something approaching a natural system. The above derivation delineates the necessary assumptions, so the conditions under which the wasteload allocation developed here should be used become clear. They may be summarized as follows:

1. The effluent flow and concentration must remain constant for extended periods of time (at least longer than the travel time from the source to the point of maximum concentration on the mixing zone boundary)

2. The mean ambient flow must remain constant for this same travel time, or for the time required to time average the effects of turbulent eddies.

3. The pollutant must be neutrally buoyant.

4. No large eddies or whirlpools which create a persistent transverse current can be tolerated.

5. The flow volume allocated to the mixing zone must be sufficiently large so that the maximum concentration in the mixing zone boundary occurs relatively far downstream.

6. The flow volume allocated to the mixing zone must be sufficiently small so that reflection from the far bank is not significant.

7. No flowing tributaries can enter the reach between the source and the point of maximum concentration on the mixing zone boundary.

8. The depth and velocity of the receiving stream must remain fairly constant in the reach between the source and the point of maximum applicable concentration. Therefore, the wasteload allocation will be more accurate when applied to low gradient receiving streams than to turbulent mountain streams.

9. Conservation of mass requires that the pollutant does not change form chemically or volitilize during travel from the source to the location of maximum concentration on the mixing zone boundary.

10. The discharge must approximate a continuous point source located at the bank.

11. The background concentration must be less than the water quality standard. If this is not the case, then the effluent concentration should be set equal to the standard.

12. The background concentration must be constant throughout the region between the point source and the point of maximum concentration on the mixing zone boundary.

The more the effluent discharge/receiving stream system conforms to the above assumptions, the better the chances that the wasteload allocation will result in a maximum concentration on the mixing zone boundary which equals the numerical criterion in the water quality standard. The permitter will have to decide whether or not this wasteload allocation yields the best permit on a case by case basis.

Several additional assumptions were made during this derivation. It is not feasible to routinely assess their validity; so they were omitted from the above list. An example of this type of assumption is that the concentration distribution is normal. There is no way to determine the validity of this assumption for a specific stream without doing an in-stream survey.

Many states require that one quarter of the flow volume be reserved for a mixing zone. In this case,

$$Q_M \equiv \frac{Q_u + Q_e}{4} .$$

Substitution in (51) yields

$$\frac{C_e}{C'} = .5165 \left(\frac{1 + Q_e^*}{Q_e^*}\right) \tag{52}$$

but $C_e \geq C$. This will be called the mixing zone wasteload allocation.

Figure 1 shows plots of C_e/C' vs. $Q_e{}^*$. The solid line represents the mixing zone wasteload allocation. Comparing this with the wasteload allocation based on the mass balance assumption (shown by the dotted line), as $Q_e{}^*$ approaches 1 (the effluent flow, Q_e, approaches the size of the upstream dilution flow Q_α), the mass balance approach yields a wasteload allocation about twice that allowed by the mixing zone method. Figure 1 also shows that when the effluent flow is much smaller than the upstream flow, as it sometimes is, the mixing zone allocation yields a much more stringent effluent concentration than the mass balance allocation does. At $Q_e{}^* = 0.03$, the mixing zone allocation yields $C_e/C' = 17.73$, and the mass balance allocation yields $C_e/C' = 34.33$.

A comparison of equations (52) and (2) shows that the ratio of the mixing zone allocation to the mass balance allocation is, of course, .5165. This ratio will hold wherever $Q_e{}^*$ is small enough so that (52) yields C_e less than C, the background concentration is zero, and one quarter of the flow volume is reserved for a mixing zone.

The reason the mixing zone allocation is more stringent than the mass balance allocation is because it does not allow the maximum concentration to exceed the numerical criterion outside the mixing zone, while mass balance only insures that standards will not be exceeded in the region where the concentration distribution is mixed uniformly across the stream. On large streams uniform mixing may not be achieved for several miles below the discharge. Therefore, the mass balance allocation allows standards to be violated over large portions of some streams.

Chapter 5
Conclusions

The mixing zone wasteload allocation incorporates the desirable features of the mass balance approach. The allocation accounts for the dilution capacity of the receiving stream, while requiring a minimum of input data. Accounting for the dilution capacity insures that the receiving stream will be protected without causing undue economic hardship to the discharger.

Input data is kept to a minimum so that special data collection trips are not required to perform the allocation. There is no need to calibrate the model for a receiving stream.

This wasteload allocation eliminates the most glaring deficiency of the mass balance allocation, which is its inability to protect water quality standards everywhere outside the mixing zone. The mass balance allocation assures that standards will not be exceeded only after complete mixing in the receiving stream has occurred.

Ease of application combined with the apparent validity of the derivation presented should persuade permitting agencies to use the mixing zone wasteload allocation. However, not all the assumptions used can be fully justified. The validity of the assumptions should be verified in the field. This may be accomplished by finding the maximum concentration on the mixing zone boundary and comparing it with the downstream concentration, C, obtained from equation (51). If the assumptions used in deriving (51) are valid, then the predicted and observed concentrations will be comparable.

The mixing zone wasteload allocation has been incorporated in the Oklahoma Water Quality Management Plan and is used routinely by the Oklahoma Water Resources Board in its permitting activities, when it is applicable. The use of a viable wasteload allocation greatly streamlines the permitting proceedure.

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